Chapter 8
Similarity

Section 1
Ratio and Proportion

GOAL 1: Computing Ratios

If *a* and *b* are two quantities that are measured in the *same* units, then the ratio of a to b is a/b. The ratio of *a* to *b* can also be written as a:b. Because a ratio is a quotient, its denominator cannot be zero.

Ratios are usually expressed in simplified form. For instance, the ratio of 6:8 is usually simplified as 3:4.

Example 1: Simplifying Ratios

1m = 100 cm

a. $\frac{12\text{ cm}}{4\text{ m}} \rightarrow \frac{12\text{ cm}}{400\text{ cm}}$

$\frac{12 \div 4}{400 \div 4} \rightarrow \frac{3}{100}$

1 ft = 12 in

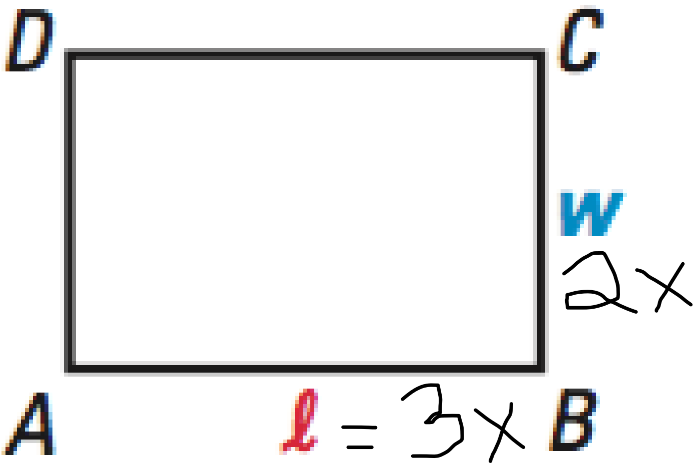
b. $\frac{6\text{ ft}}{18\text{ in.}} \rightarrow \frac{72\text{ in}}{18\text{ in}}$

$\frac{72 \div 6}{18 \div 6} \rightarrow \frac{12 \div 3}{3 \div 3} \rightarrow \frac{4}{1}$

Example 2: Using Ratios

$$P = 2\ell + 2w$$

The perimeter of rectangle ABCD is 60 centimeters. The ratio of AB:BC is 3:2. Find the length and width of the rectangle.



$$60 = 2(3x) + 2(2x)$$

$$60 = 10x$$

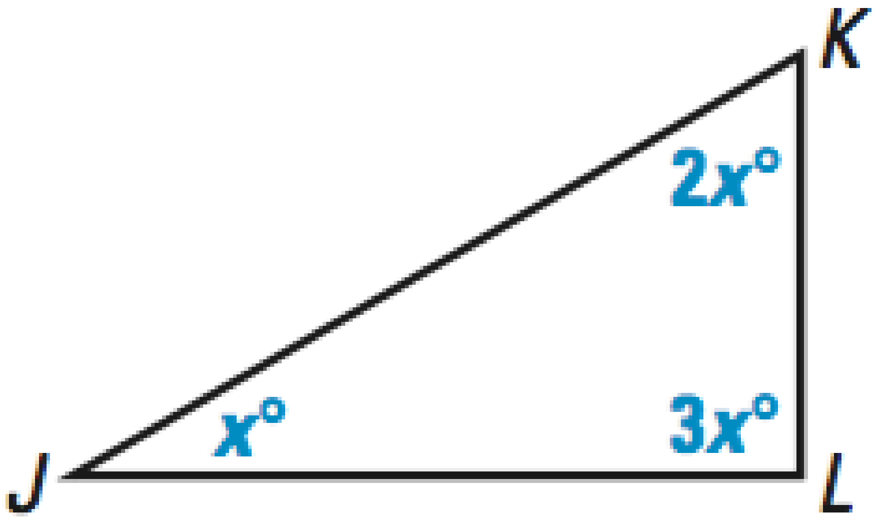
$$6 = x$$

$$\ell \rightarrow 3x \rightarrow 3(6) = 18 \text{ cm}$$

$$w \rightarrow 2x \rightarrow 2(6) = 12 \text{ cm}$$

Example 3: Using Extended Ratios

The measure of the angles in ΔJKL are in the *extended ratio* of 1:2:3. Find the measures of the angles.



$$1x + 2x + 3x = 180$$

$$6x = 180$$

$$x = 30$$

$$\angle J \rightarrow x \rightarrow 30^\circ$$

$$\angle K \rightarrow 2x \rightarrow 2(30) = 60^\circ$$

$$\angle L \rightarrow 3x \rightarrow 3(30) = 90^\circ$$

Example 4: Using Ratios

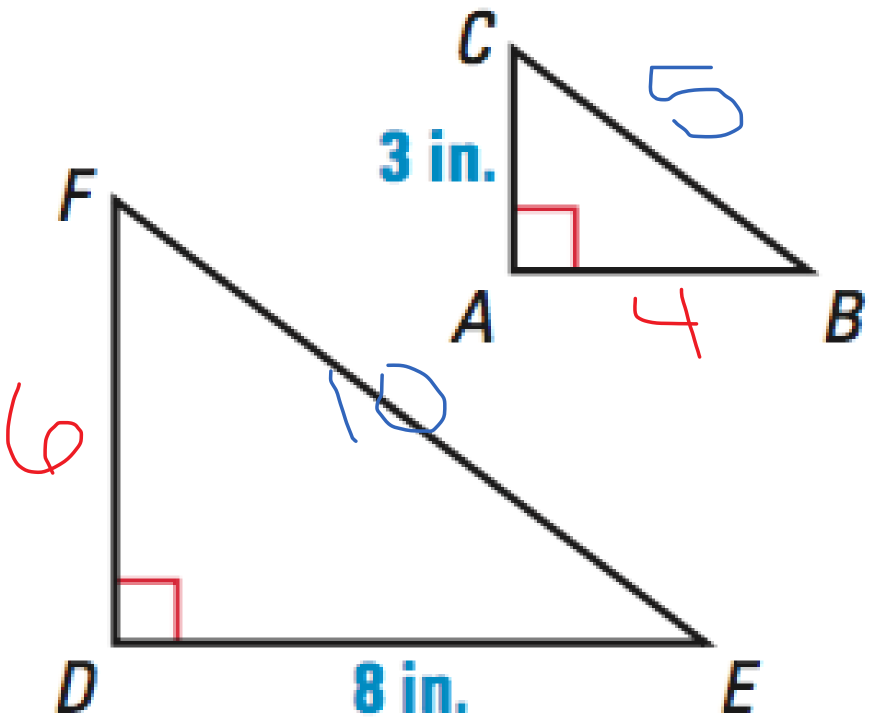
The ratios of the side lengths of ΔDEF to the corresponding side lengths of ΔABC are 2:1. Find the unknown lengths.

$$3^2 + 4^2 = c^2$$

$$9 + 16 = c^2$$

$$\sqrt{25} = \sqrt{c^2}$$

$$5 = c$$



GOAL 2: Using Proportions

An equation that equates two ratios is a **proportion.** For instance, if the ratio $\frac{a}{b}$ is equal to the ratio $\frac{c}{d}$, then the following proportion can be written:

Means Extremes

$$\frac{a}{b} = \frac{c}{d}$$

The numbers a and d are the **extremes** of the proportion. The numbers b and c are the **means** of the proportion.

PROPERTIES OF PROPORTIONS

1. **CROSS PRODUCT PROPERTY** The product of the extremes equals the product of the means.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } ad = bc.$$

2. **RECIPROCAL PROPERTY** If two ratios are equal, then their reciprocals are also equal.

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{b}{a} = \frac{d}{c}.$$

Example 5: Solving Proportions

a. $\frac{4}{x} = \frac{5}{7}$

$5 \cdot x = 4 \cdot 7$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{28}{5}$

$x = 5.6$

b. $\frac{3}{y+2} = \frac{2}{y}$

$3 \cdot y = 2(y+2)$

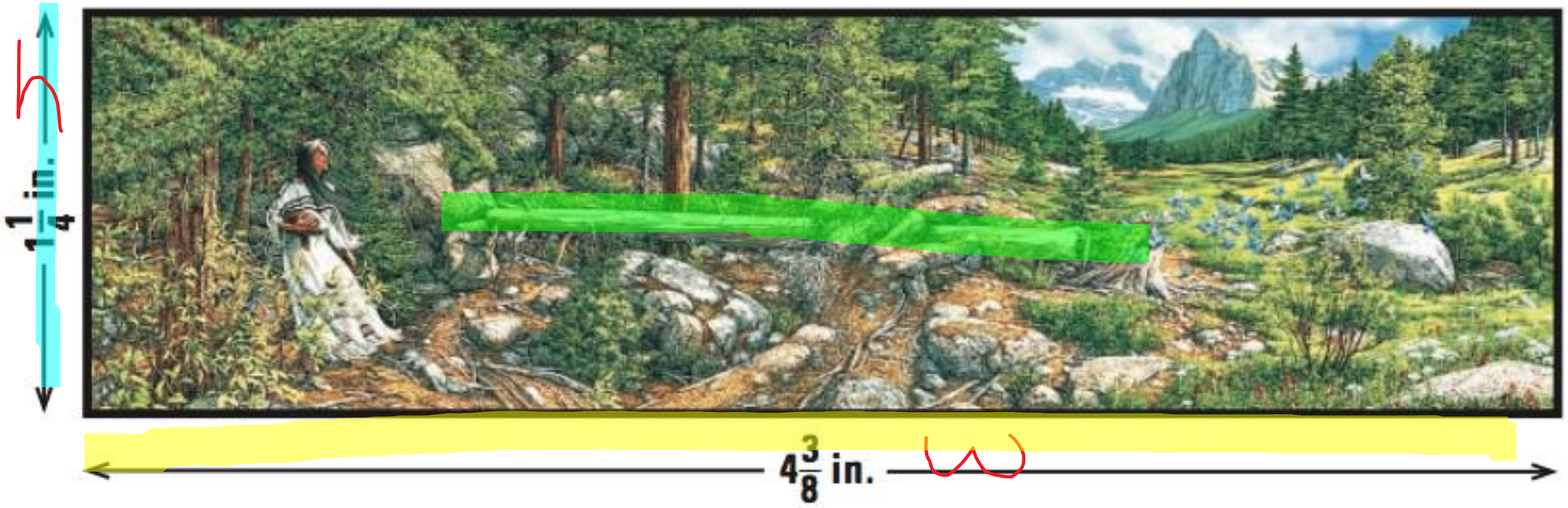
$3y = 2y + 4$

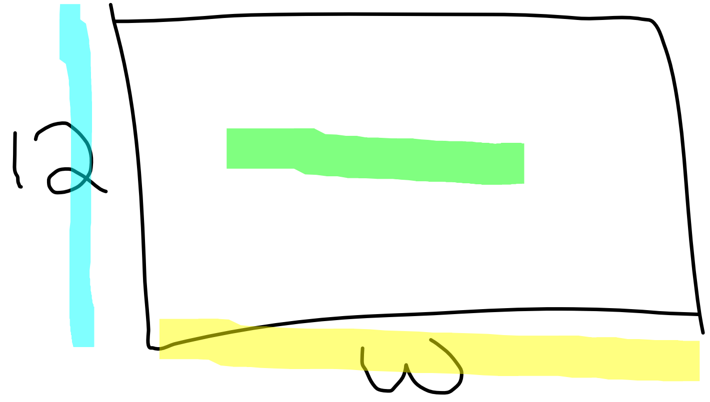
$-2y \quad -2y$

$y = 4$

Example 6: Solving a Proportion

Painting: The photo shows Bev Doolittle's paint *Music in the Wind.* Her actual painting is 12 inches high. How wide is it?





$$\frac{w}{4.375} = \frac{12}{1.25}$$

$$\frac{\cancel{1.25}w}{1.25} = \frac{52.5}{1.25} \rightarrow w = 42 \text{ in.}$$

Example 7: Solving a Proportion

Estimate the length of the hidden flute in Bev Doolittle's actual painting.

(Note: in the photo, the flute is 1 7/8 inches long)

$$\frac{1.25}{12} = \frac{1.875}{x}$$

$$\frac{1.25x}{1.25} = \frac{22.5}{1.25}$$

$$x = 18 \text{ in.}$$

EXIT SLIP